Teacher notes
Topic A

Average power

In many problems we will be asked to find the average power developed by a force. This can be done simply by calculating the amount ΔE by which the total energy of the system changed and dividing by the time Δt taken: $\bar{P}=\frac{\Delta E}{\Delta t}$.

An alternative is to use the fact that at any instant the power developed by a force F is $P=Fv$ where v is the instantaneous speed. To find the average power we need to find the average of the product Fv. It is shown below that this is given by

$$\bar{P}=\bar{F}\times\frac{u+v}{2}$$

where u is the initial speed and $\bar{F}$ is the average **net** force. (Notice that $\frac{u+v}{2}$ is NOT the average speed; it is the average speed only when the acceleration is constant. This means that, in general, you **cannot** write $\bar{P}=\bar{F}\times\bar{v}$.)

$$\begin{aligned}\bar{P}&=\frac{1}{T}\int_0^T Fv\,dt\\&=\frac{1}{T}\int_0^T m\frac{dv}{dt}v\,dt=\frac{1}{T}\int_0^T mv\,dv\\&=\frac{1}{T}\left(\frac{1}{2}mv^2-\frac{1}{2}mu^2\right)=\frac{1}{2T}m(v-u)(v+u)\\&=(m\frac{\Delta v}{T})\times(\frac{u+v}{2})\\&=\bar{F}\frac{u+v}{2}\end{aligned}$$

It is stressed that $\bar{P}=\bar{F}\times\frac{u+v}{2}$ gives the average power developed by the **net** force not just any force.

Now imagine a rope raising a mass m vertically upwards. What is the average power developed by the tension T in the rope? Assume that the tension is constant.

$$F_{net}=T-mg\Rightarrow T=F_{net}+mg$$

The average power developed by the tension is then

$$\bar{P} = \bar{F}_{\text{net}} \times \frac{u+v}{2} + \frac{mgs}{t}$$

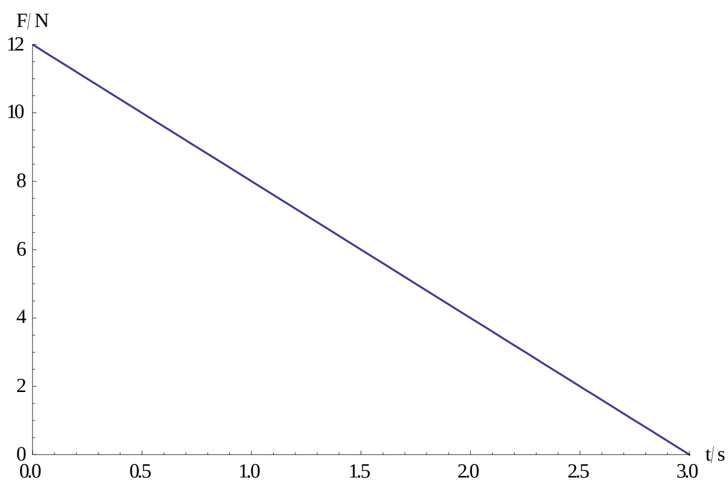
$$= \bar{F}_{\text{net}} \times \frac{u+v}{2} + mg \times \frac{\frac{u+v}{2}t}{t} \qquad \text{(acceleration is constant so } s = \frac{u+v}{2}t\text{)}$$

$$= (F_{\text{net}} + mg) \times \frac{u+v}{2} \qquad \text{(we do not need the average since the net force is constant)}$$

$$= T \times \frac{u+v}{2}$$

This means that *when the acceleration is constant*, we can still use $\bar{P} = F \times \frac{u+v}{2}$ for the average power of *any* constant force F.

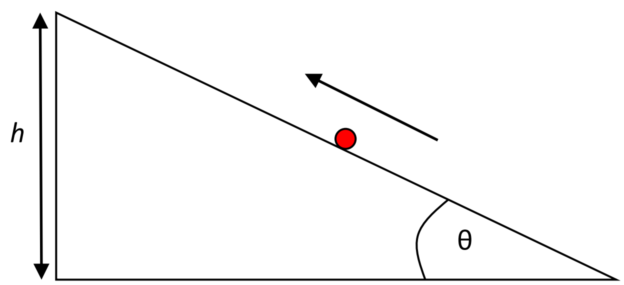
As an example, consider the average power developed by a force acting on a body of mass 2.0 kg that is initially at rest. The force that varies with time as shown:



Method 1: The impulse is $\frac{12 \times 3.0}{2} = 18$ N s and so the final speed is $v = \frac{18}{2.0} = 9.0 \text{ m s}^{-1}$ The change in total energy is the change in kinetic energy $\Delta E = \frac{1}{2} \times 2.0 \times 9.0^2 = 81$ J and dividing by the time to get $\bar{P} = \frac{81}{3.0} = 27$ W.

Method 2: The impulse is $\frac{12 \times 3.0}{2} = 18$ N s and so the average force is $\bar{F} = \frac{18}{3} = 6.0$ N. The final speed is $v = \frac{18}{2.0} = 9.0 \text{ m s}^{-1}$ and so $\bar{P} = 6.0 \times \frac{0+9.0}{2} = 27$ W.

Consider now a constant force *F* that moves a body of mass *m* = 4.0 kg up an inclined plane that makes an angle $\theta = 30^\circ$ to the horizontal raising the mass a vertical height *h* = 8.0 m. The body moves at constant speed $v = 4.0\ \text{m s}^{-1}$. What is the average power developed by *F*?



Method 1: The change in potential energy is $mgh = 4.0 \times 10 \times 8.0 = 320\ \text{J}$ and this change occurs in a time of (the distance travelled along the plane is 16 m) $t = \frac{16}{4.0} = 4.0\ \text{s}$. There is no change in kinetic energy.

So $\bar{P} = \frac{\Delta E}{\Delta t} = \frac{320}{4.0} = 80\ \text{W}$.

Method 2: The net force is zero so $F - mg\sin\theta = 0 \Rightarrow F = mg\sin\theta = 4.0 \times 10 \times \frac{1}{2} = 20\ \text{N}$. There is no acceleration so $\bar{P} = F \times \frac{u+v}{2} = 20 \times \frac{4.0+4.0}{2} = 80\ \text{W}$.

Suppose now that the force *F* in the previous example is a constant force of 28 N.

What is the average power developed by *F*?

We can do this in two ways. For both methods we will need:

The net force is $F - mg\sin\theta = 28 - 20 = 8.0\ \text{N}$ up the plane and so the acceleration is $2.0\ \text{m s}^{-2}$. Then (the distance travelled along the plane is 16 m) $v^2 = 2as = 2 \times 2.0 \times 16 = 64 \Rightarrow v = 8.0\ \text{m s}^{-1}$.

Method 1: At the top of the incline the mass gained potential energy $mgh = 4.0 \times 10 \times 8.0 = 320\ \text{J}$. The change in kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2} \times 4.0 \times 8.0^2 = 128\ \text{J}$. The change in total energy is then

$320+128=448\text{ J}$. The time to get to the top is given by $v=at \Rightarrow t=\frac{8.0}{2.0}=4.0\text{ s}$. Hence,

$\bar{P}=\frac{\Delta E}{\Delta t}=\frac{448}{4.0}=112\text{ W}$.

Method 2: The acceleration is constant so we may use $\bar{P}=F\frac{u+v}{2}=28\times\frac{0+8.0}{2}=112\text{ W}$.

Class example: A constant net force of 6.0 N accelerates a body from a speed of 2.0 m s^{-1} to a speed of 12 m s^{-1}. What is the average power developed?

We use $\bar{P}=F\times\frac{u+v}{2}=6.0\times\frac{2.0+12}{2}=42\text{ W}$.

How would you use $\bar{P}=\frac{\Delta E}{T}$ here since you don't know the mass?

($\Delta E=\frac{1}{2}m\times(12^2-2.0^2)=70m$.

$12=2.0+aT=2.0+\frac{F}{m}T=2.0+\frac{6.0}{m}T$, i.e. $10=\frac{6.0}{m}T$, so $\frac{m}{T}=0.60$.

Hence, $\bar{P}=\frac{\Delta E}{T}=\frac{70m}{T}=70\times0.60=42\text{ W}$.)

Summary

For constant acceleration we can use $\bar{P}=F\times\frac{u+v}{2}$ for **any constant** force *F*.

For non-constant acceleration we can use $\bar{P}=\bar{F}\times\frac{u+v}{2}$ for a **net** force *F*.